NOMBRES - Curiosités, théorie et usages

Dérivées - Formulaire

Retour Page Principale

Polynôme	Fonction dérivée
$y = a$ $y = a \cdot x$ $y = a \cdot x^2$ $y = a \cdot x^3$ $y = a \cdot x^r$	$y' = 0$ $y' = a$ $y' = 2a \cdot x$ $y' = 3a \cdot x^2$ $y' = a \cdot r \cdot x^{r-1}$
$y = x$ $y = ax + b$ $y = ax^2 + bx + c$ $y = ax^3 + bx^2 + cx + d$ $y = ax^4 + bx^3 + cx^2 + dx + e$	$y' = 1$ $y' = a$ $y' = 2ax + b$ $y' = 3ax^2 + bx + c$ $y' = 4ax^3 + 3bx^2 + cx + d$
$y = (ax + b)^2$ $= a^2x^2+2abx+b^2$	$y' = 2a\,(ax + b)$ $= 2a^2x + 2ab$
$y = (ax + b)^3$ $= a^3x^3+3a^2bx^2+ab^2x+b^3$	$y' = 3a\,(ax + b)^2$ $= 3a^3x^2+6a^2bx+3ab^2$
$y = (ax + b)\,(cx + d)$	$y' = 2acx + ad + bc$

Haut

Inverse (x au dénominateur)	Fonction dérivée
$y = \frac{1}{x}$ $z = \frac{a}{b \cdot x}$ $y = \frac{1}{u}$ $z = \frac{u}{v}$	$y' = -\frac{1}{x^2}$ $z' = -\frac{a}{b \cdot x^2}$ $y' = -\frac{u'}{u^2}$ $z' = \frac{u' \cdot v - u \cdot v'}{v^2}$
$\frac{a x^3 + b x^2 + c x + d}{x}$	$\frac{3 a x^2 + 2 b x + c}{x} - \frac{a x^3 + b x^2 + c x + d}{x^2}$
$\frac{a x^3 + b x^2 + c x + d}{x^2}$	$\frac{3 a x^2 + 2 b x + c}{x^2} - \frac{2 (a x^3 + b x^2 + c x + d)}{x^3}$
$\frac{a x^3 + b x^2 + c x + d}{e x^2 + d}$	$\frac{3 a x^2 + 2 b x + c}{e x^2 + d} - \frac{2 (a x^3 + b x^2 + c x + d) e x}{(e x^2 + d)^2}$
$\frac{k}{a x^2 + b x + c}$	$-\frac{k (2 a x + b)}{(a x^2 + b x + c)^2}$
$\frac{k x + h}{a x^2 + b x + c}$	$\frac{k}{a x^2 + b x + c} - \frac{(k x + h)(2 a x + b)}{(a x^2 + b x + c)^2}$
$\frac{k x^2 + h x + l}{a x^2 + b x + c}$	$\frac{2 k x + h}{a x^2 + b x + c} - \frac{(k x^2 + h x + l)(2 a x + b)}{(a x^2 + b x + c)^2}$

Haut

Trigonométrie	Fonction dérivée
$y = \sin x$ $y = \sin(ax^2+bx+c)$	$y' = \cos x$ $y' = (2ax+b)\cos(ax^2+bx+c)$
$y = \cos x$ $y = \cos(ax^2+bx+c)$	$y' = -\sin x$ $y' = -(2ax+b)\sin(ax^2+bx+c)$
$y = \tan x$ $y = \tan(ax^2+bx+c)$	$y' = 1 + tg^2 x = \frac{1}{\cos^2 x} = \sec^2(x)$ $y' = (2ax+b)(1 + \tan^2(ax^2+bx+c))$
$y = \cot x$	$y' = -1 - \cot^2 x)$
$y = \arcsin x = \sin^{-1}(x)$	$y' = \frac{1}{\sqrt{1-x^2}}$
$y = \arccos x = \cos^{-1}(x)$	$y' = \frac{-1}{\sqrt{1-x^2}}$
$y = \arctan x = \tan^{-1}(x)$	$y' = \frac{1}{1+x^2}$
$y = \text{arccot}\, x = \cot^{1}(x)$	$y' = \frac{-1}{1+x^2}$
$y = \sec(x) = 1/\cos(x)$	$y' = \sec(x)\tan(x)$ $= \frac{\sin(x)}{\cos^2(x)} = \frac{\sin(x)}{1-\sin^2(x)}$
$y = \csc(x) = 1/\sin(x)$	$y' = -\csc(x)\,\text{cotan}(x)$ $= \frac{-\cos(x)}{\sin^2(x)} = \frac{-\cos(x)}{1-\cos^2(x)}$
$y = \sinh(x)$ $y = \cosh(x)$ $y = \tanh(x)$ $y = \text{csch}(x)$	$y' = \cosh(x)$ $y' = \sinh(x)$ $y' = \text{sech}^2(x)$ $y' = -\text{csch}^2(x)$

Haut

Trigonométrie (fonctions)	Fonction dérivée
$y = \sin\left(\frac{x}{2}\right)$ $z = \cos\left(\frac{x}{2}\right)$	$y' = \frac{\cos(x/2)}{2} \quad z' = -\frac{\sin(x/2)}{2}$
$y = \sin^2 x$ $y = \cos^2 x$	$y' = 2\sin(x) \cdot \cos(x)$ $y' = -2\sin(x) \cdot \cos(x)$
$y = \sin^2(x^2)$ $y = \sin^3(x^2)$	$y' = 4x \cdot \sin(x^2) \cdot \cos(x^2)$ $y' = 6x \cdot \sin^2(x^2) \cdot \cos(x^2)$
$y = \sin^3 x$ $y = \cos^3 x$	$y' = 3\sin^2(x) \cdot \cos(x)$ $y' = -3\sin(x) \cdot \cos^2(x)$
$y = \sin\left(\frac{1}{x}\right)$ $z = \cos\left(\frac{1}{x}\right)$	$y' = \frac{\cos(1/x)}{x^2} \quad z' = \frac{\sin(1/x)}{x^2}$
$y = x \cdot \sin\left(\frac{1}{x}\right)$ $y = x \cdot \cos\left(\frac{1}{x}\right)$	$y' = \sin\left(\frac{1}{x}\right) - \frac{1}{x}\cos\left(\frac{1}{x}\right)$ $y' = \cos\left(\frac{1}{x}\right) + \frac{1}{x}\sin\left(\frac{1}{x}\right)$
$y = x^2 \cdot \sin\left(\frac{1}{x}\right)$ $y = x^2 \cdot \cos\left(\frac{1}{x}\right)$	$y' = 2x \cdot \sin\left(\frac{1}{x}\right) - \cos\left(\frac{1}{x}\right)$ $y' = 2x \cdot \cos\left(\frac{1}{x}\right) + \sin\left(\frac{1}{x}\right)$
$y = \sin(\sqrt{x})$ $z = \cos(\sqrt{x})$	$y' = \frac{\cos(\sqrt{x})}{2\sqrt{x}} \quad z' = -\frac{\sin(\sqrt{x})}{2\sqrt{x}}$
$y = \sin^3(x)$ $+ \cos^3(x)$	$y' = 3\sin^2(x) \cdot \cos(x)$ $- 3\cos^2(x) \cdot \sin(x)$
$y = \tan\left(\frac{x}{2}\right) - \cot\left(\frac{x}{2}\right)$	$y' = \frac{2}{\sin^2(x)}$
$y = \sin(\sin(x))$ $y = \sin(\cos(x))$ $y = \cos(\sin(x))$ $y = \cos(\cos(x))$	$y' = \cos(x) \cdot \cos(\sin(x))$ $y' = -\sin(x) \cdot \cos(\cos(x))$ $y' = -\cos(x) \cdot \sin(\sin(x))$ $y' = \sin(x) \cdot \sin(\cos(x))$

Haut

Logarithmes	Fonction dérivée
$y = \ln x$ $z = \log_a(x)$	$y' = \frac{1}{x}$ $z' = \frac{1}{x\ln(a)}$ $a > 0, c \neq 1$
$y = \ln(u)$	$y' = \frac{u'}{u}$
$y = \log_a(u)$	$y' = \frac{u'}{u \cdot \ln(a)}$
$y = \ln(ax^2+bx+c)$	$y' = \frac{2ax+b}{ax^2+bx+c}$
$y = (\ln x)^2$ $z = \ln(x^2)$	$y' = \frac{2\ln(x)}{x}$ $z' = \frac{2}{x}$
$y = (\ln x)^3$ $z = \ln(x^3)$	$y' = \frac{3(\ln(x))^2}{x}$ $z' = \frac{3}{x}$
$y = \ln u$	$y' = \frac{u'}{u}$ $(u > 0)$
$y = \ln\sqrt{\frac{1-\sin(x)}{1-\sin(x)}}$	$y' = -\sec(x)$

Haut

Exponentielle	Fonction dérivée
$y = k^x$	$y' = k^x . \ln(k)$
$y = k^{ax}$	$y' = a \cdot k^{ax} \cdot \ln(k)$
$y = k^{ax^2+bx+c}$	$y' = (2ax+b)\, k^{ax^2+bx+c}\, \ln(k)$
$y = e^{a \cdot x}$	$y' = a \cdot e^x$
$y = a \cdot e^{bx}$	$y' = a \cdot b\, e^{bx}$
$y = a \cdot e^{bx^2+c}$	$y' = 2a \cdot b\, e^{bx^2+c}$
$y = a^u$	$y' = u' \cdot \ln(a) \cdot a^u$
$y = e^u$	$y' = u' \cdot e^u$
$y = u^a$	$y' = u' \cdot a \cdot u^{a-1}$
$y = x^x$	$y' = x^x\,(1 + \ln(x))$
$y = b \cdot x^{a \cdot x}$	$y' = a \cdot b \cdot x^{a \cdot x}\,(1 + \ln(x))$
$y = e^{-x^n}$	$y' = -n \cdot x^{n-1} \cdot e^{-x^n}$
$y = e^{-\frac{1}{x^n}}$	$y' = n \cdot x^{-n-1} \cdot e^{-x^{-n}}$
$y = (ax^2 + bx + c)^{d^e}$	$y' = d^e (ax^2 + bx + c)^{d^e} \dfrac{2ax + b}{ax^2 + bx + c}$

Haut

Racines	Fonction dérivée
$y = a\sqrt{x} = a \cdot x^{\frac{1}{2}}$ $y = a\sqrt[3]{x} = a \cdot x^{\frac{1}{3}}$ $y = a\sqrt[q]{x} = a \cdot x^{\frac{1}{q}}$ $y = a\sqrt[q]{x^p} = a \cdot x^{\frac{p}{q}}$ $y = a\sqrt[3]{x^2} = a \cdot x^{\frac{p}{q}}$	$y' = \frac{a}{2} \cdot x^{-1/2} = \frac{a}{2x^{1/2}} = \frac{a}{2\sqrt{x}}$ $y' = \frac{a}{3} \cdot x^{-2/3} = \frac{a}{3x^{2/3}} = \frac{a}{3(\sqrt{x})^2}$ $y' = \frac{a}{q} \cdot x^{\frac{1}{q}-1}$ $y' = \frac{a \cdot p}{q} \cdot x^{\frac{p}{q}-1}$ $y' = \frac{2a}{3} \cdot x^{-1/3}$
$y = \sqrt{1+x^2}$ $z = \sqrt{1-x^2}$	$y' = \frac{x}{\sqrt{1+x^2}}$ $\quad z' = -\frac{x}{\sqrt{1-x^2}}$
$y = \sqrt{1-x^2} \cdot \sqrt{1+x^2}$	$y' = -\frac{2x^3}{\sqrt{1-x^2} \cdot \sqrt{1+x^2}}$
$y = \sqrt[3]{1-x^2}$	$y' = -\frac{2x}{3(1-x^2)^{2/3}}$
$y = \sqrt[3]{1-x^3}$	$y' = -\frac{x^2}{(1-x^2)^{2/3}}$
$k\sqrt{ax^2+bx+c}$	$\frac{k(2ax+b)}{2\sqrt{ax^2+bx+c}}$
$(ax^2+bx+c)^{1/3}$	$\frac{2ax+b}{3(ax^2+bx+c)^{2/3}}$
$\left(\frac{ax^2+bx+c}{kx^2+hx+l}\right)^{1/3}$	$\frac{\frac{2ax+b}{kx^2+hx+l} - \frac{(ax^2+bx+c)(2kx+h)}{(kx^2+hx+l)^2}}{3\left(\frac{ax^2+bx+c}{kx^2+hx+l}\right)^{2/3}}$

Haut

Dérivée des fonctions composées

Fonction	Dérivée
$y = u + v$ $y = u \cdot v$ $y = u \cdot v \cdot w$	$y' = u' + v'$ $y' = u \cdot v' + u' \cdot v$ $y' = u' \cdot v \cdot w + u \cdot v' \cdot w + u \cdot v \cdot w'$
$y = \frac{1}{v} \quad z = \frac{u}{v}$	$y' = -\frac{v'}{v^2} \quad z' = \frac{u' \cdot v - u \cdot v'}{v^2}$
$y = u^n$	$y' = n \cdot u^{(n-1)} \cdot u'$
$y = \sqrt{u}$	$y' = \frac{u'}{2\sqrt{u}}$
$y = g(u)$ & $u = f(x)$	$y'_x = y'_u \cdot u'_x$ On écrit aussi: $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$
$y = \ln u$ $y = \ln(ax^2 + bx + c)$	$y' = \frac{u'}{u}$ $y' = \frac{2ax + b}{ax^2 + bx + c}$
$y = e^u$ $y = e^{ax^2 + bx + c}$	$y' = e^u \cdot u'$ $y' = 2a \cdot x \cdot e^{ax^2 + bx + c} + b\, e^{ax^2 + bx + c}$

Haut

Exemples

Fonction / Dérivée	
$(a x^2 + b x)(c x^3 + d x^2)$	$(2ax + b)(cx^3 + dx^2) + (ax^2 + bx)(3cx^2 + 2dx)$ $5acx^4 + (4ad + 4bc)x^3 + 3bdx^2$
$\dfrac{ax+b}{cx+d}$	$\dfrac{a}{cx+d} - \dfrac{(ax+b)c}{(cx+d)^2}$

Exemples de calcul

$$[\tan(x)]' = \left[\frac{\sin(x)}{\cos(x)}\right]' = \frac{[\sin(x)]'\cos(x) - \sin(x)[\cos(x)]'}{\cos^2(x)}$$
$$= \frac{\cos(x)\cos(x) - \sin(x)(-\sin(x))}{\cos^2(x)} = \frac{1}{\cos^2(x)}$$

$$[\sec(x)]' = \left[\frac{1}{\cos(x)}\right]' = -\frac{[\cos(x)]'}{\cos^2(x)} = \frac{\sin(x)}{\cos^2(x)}$$
$$= \frac{\sin(x)}{\cos(x)} \cdot \frac{1}{\cos(x)} = \tan(x)\sec(x)$$

$$[\cos(2x) - 2\sin(x)]' = [\cos(2x)]' - [2\sin(x)]'$$
$$= (-\sin(2x) \cdot [2x]' - 2[\sin(x)]' = -2\sin(2x) - 2\cos(x)$$
$$= -4\sin(x)\cos(x) - 2\cos(x) = -2\cos(x)(2\sin(x) + 1)$$

$$\left[\cos(x) + \frac{\cos^3(x)}{3}\right]' = [\cos(x)]' + \left[\frac{\cos^3(x)}{3}\right]'$$
$$= -\sin(x) - \frac{1}{3} \cdot 3\cos^2(x) \cdot [\cos(x)]'$$
$$= -\sin(x) - \cos^2(x) \cdot (-\sin(x)) = -\sin(x)(1 + \cos^2(x) \cdot \sin(x)$$
$$= -\mathrm{sinx}(x)(1 - \cos^2(x)) = \sin(x)\sin^2(x) = -\sin^3(x)$$

$$\left[\tan(x) + \frac{\tan^3(x)}{3}\right]' = [\tan(x)]' + \left[\frac{\tan^3(x)}{3}\right]'$$
$$= \frac{1}{\cos^2} + \frac{1}{3} \cdot 3\tan^2(x) \cdot [\tan(x)]'$$

$$= \frac{1}{\cos^2} + \tan^2(x) \cdot \frac{1}{\cos^2(x)}$$

$$= \frac{1 + \tan^2(x)}{\cos^2(x)} = \frac{1}{\cos^4(x)} = \sec^4(x)$$

$$\left[\frac{\sin(x)}{1 + cos(x)}\right]' = \frac{\cos(x) \cdot \left(1 + \cos(x)\right) - \text{sins}(x) \cdot \left(-\sin(x)\right)}{(1 + \cos^2(x)}$$

$$= \frac{\cos(x) + \cos^2(x) + \sin^2(x)}{(1 + \cos^2(x)} = \frac{\cos(x) + 1}{(1 + \cos^2(x)} = \frac{1}{1 + \cos(x)}$$

Haut

Liens / Référence

>>> **Table de dérivées usuelles – Wikipédia**

>>> **Tableau des dérivées, primitives et développements limités**

>>> **Calculus / Tables of Derivatives**

>>> **Derivative Rules – Maths is Fun**

>>> **Derivative – Wolfram MathWorld**

>>> **Derivatives – Calculus Cheat Sheet – Paul Dawkins**

>>> **Derivative Table – Math24.net – *Exemples de caluls***

Dérivateur en ligne

>>> **Derivative Solver – Wolfram – *Calcul de dérivées (+ info)***

>>> **Derivative calculator – *Calcul, explications, courbes***

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